A norm theorem for differential forms

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Abstract

In this talk, we let F to be a field of characteristic 2. The bijectivity [2] between the Kato-Milne cohomology group $H_2^{n+1}(F)$ and the Witt group $W_q(F)$ enables us to translate certain results from the frame of quadratic forms to differential forms and vice verse. The cohomology group $H_2^{n+1}(F)$ is the cokernel of the Artin-Schreier operator $\wp : \Omega_F^n \to \Omega_F^n/\mathrm{d}\Omega_F^{n-1}$ given by $x\frac{\mathrm{d}x_1}{x_1} \wedge \cdots \wedge \frac{\mathrm{d}x_n}{x_n} \mapsto (x^2-x)\frac{\mathrm{d}x_1}{x_1} \wedge \cdots \wedge \frac{\mathrm{d}x_n}{x_n} + \mathrm{d}\Omega_F^{n-1}$, where Ω_F^n is the space of n- differential forms over F and $\mathrm{d}\Omega_F^{n-1}$ is the group of exact differential forms. We call a differential form $w \in \Omega_F^n$ to be *hyperbolic* if $w \in \mathrm{d}\Omega_F^{n-1} + \wp(\Omega_F^n)$ and $\alpha \in F^*$ a *norm* of ω if $w \wedge \frac{\mathrm{d}\alpha}{\alpha}$ is hyperbolic. A recent work by Aravire, Laghribi and O'Ryan [1] invoked our interest to ask for an analogue of Knebush's norm theorem [3, Theorem 4.2] in the setting of differential forms. For this our inspiration being a recent result by Laghribi and Mukhija [4, Theorem 1.1] which completes the norm theorem for quadratic forms in characteristic 2 using Scharlau's transfer. Inspired from these results, we establish the norm theorem for differentials forms which states the following:

Theorem 1. *Suppose that F is a field of characteristic 2. Let $w \in \Omega_F^m$, $\rho \in F[x_1, \ldots, x_n]$ be a normed irreducible inseparable polynomial and $K = F(x_1, \ldots, x_n)$. Then, the following two statements are equivalent:*

1. *w is hyperbolic over $F(\rho)$.*
2. *ρ is a norm of w_K.*

In addition, we also prove the above theorem when $\rho(x) \in F[x]$ is an irreducible polynomial such that the extension $F(\rho)/F$ is Galois. In this talk, we will cover the basic terminologies and briefly give the sketch of proof when K/F is a purely inseparable simple extension.

References

[1] Aravire R., Laghribi A., O'Ryan M., *Transfer for Kato-Milne cohomology over purely inseparable extensions,* Preprint 2021.

[2] Kato K., *Symmetric bilinear forms, quadratic forms and Milnor K-theory in characteristic* 2*,* Inventiones Math. **66** (1982), 493-510.

[3] Knebusch M., *Specialization of quadratic and symmetric bilinear forms, and a norm theorem,* Acta Arith., **24** (1973), 279-299.

[4] Laghribi A., Mukhija D., *On the norm theorem for semisingular quadratic forms*, J. Pure Appl. Algebra, **225** (2021)106601, 13 pp.